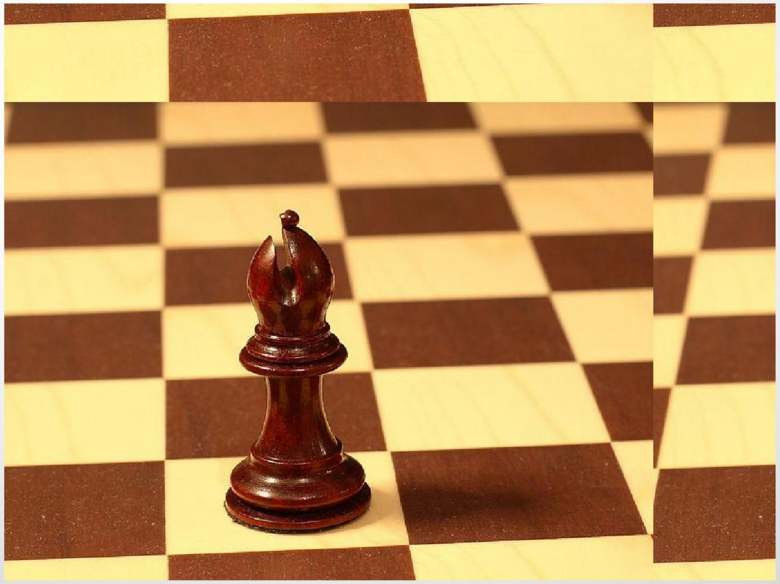
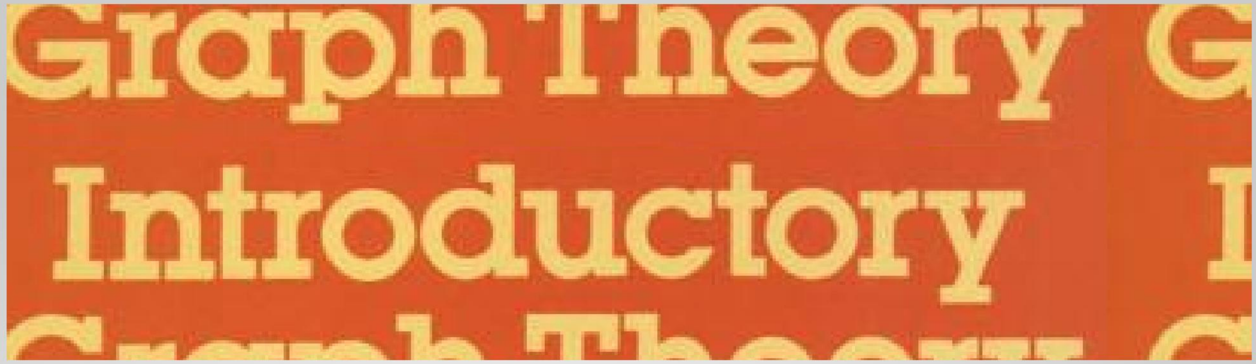
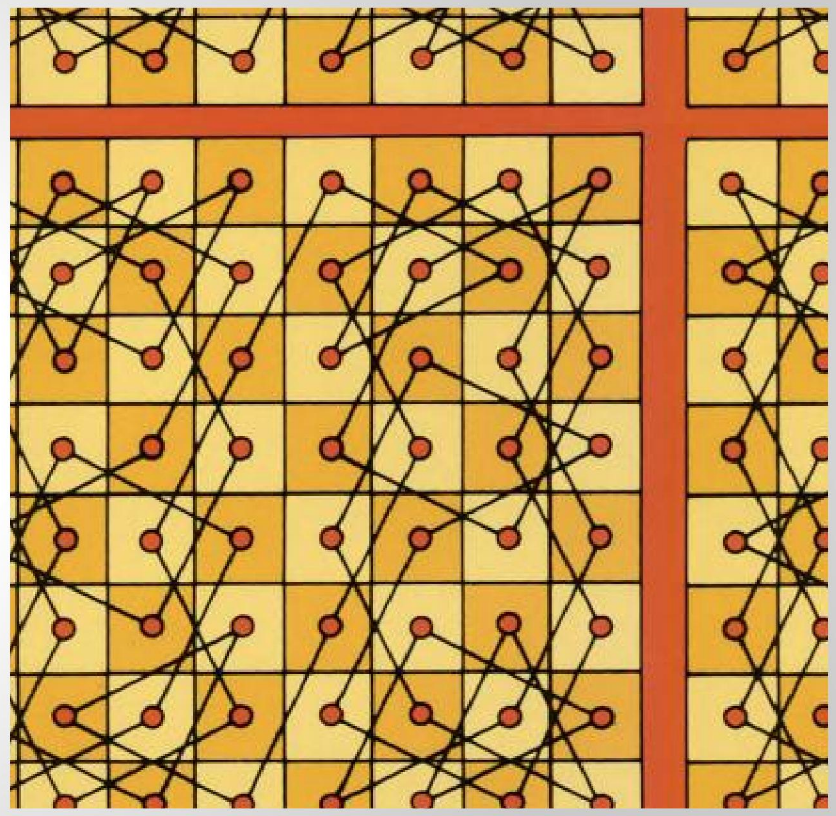
The Domination of Three Dimensional Chessboards by Bishops

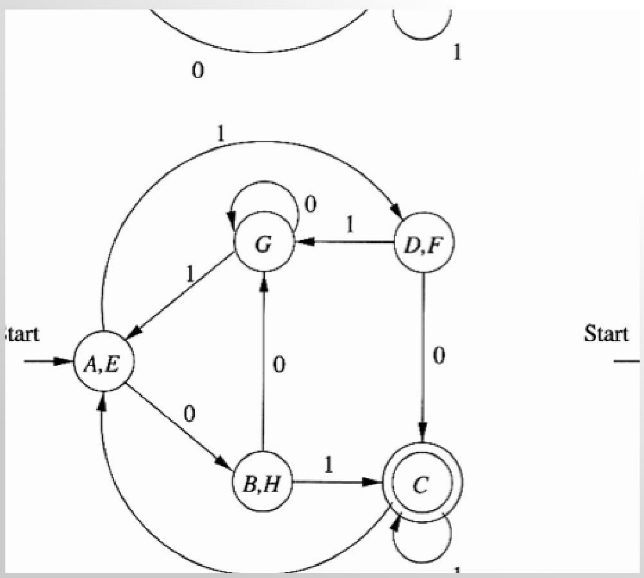
Presentation by: Joshua Eisenberg





- Chessboard domination is a problem within the area of graph theory
- Graph theory is about the relationship of vertices that are connected by edges
  - It is the foundation for the theory of computation, for example: Turing machines, Finite Automata



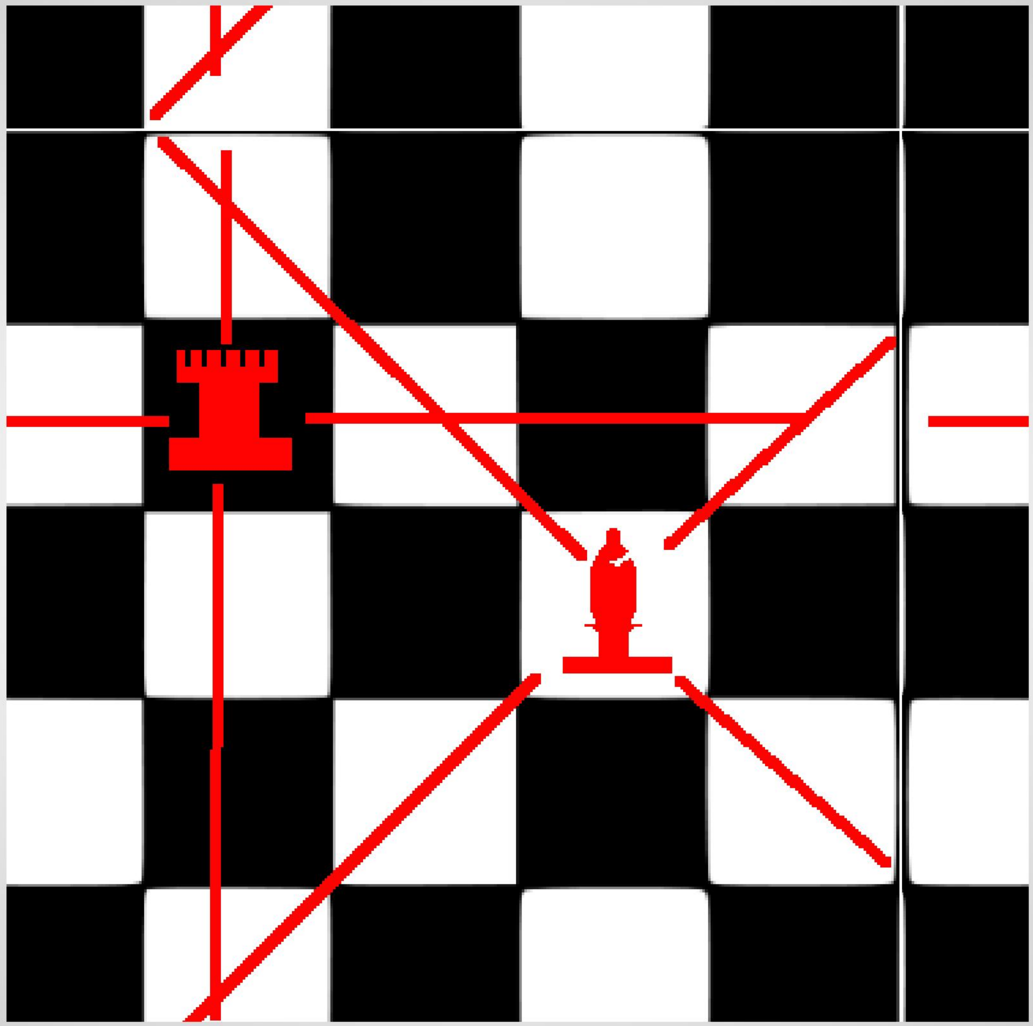


Note: To the left is a discrete finite automata and above is a traversal of an 8x8 chessboard by a knight

Graph Theory Continued

- How to to depict chessboards as graphs
  - Every square is a vertex
  - Edges are generated by the paths a particular chess piece travels from one square to another

The Motion of Bishops and Rooks



What is Chessboard domination?

bishops in order to attack every square on the chessboard. The number, $\gamma(G)$ is the size of the smallest dominating set for a graph G.

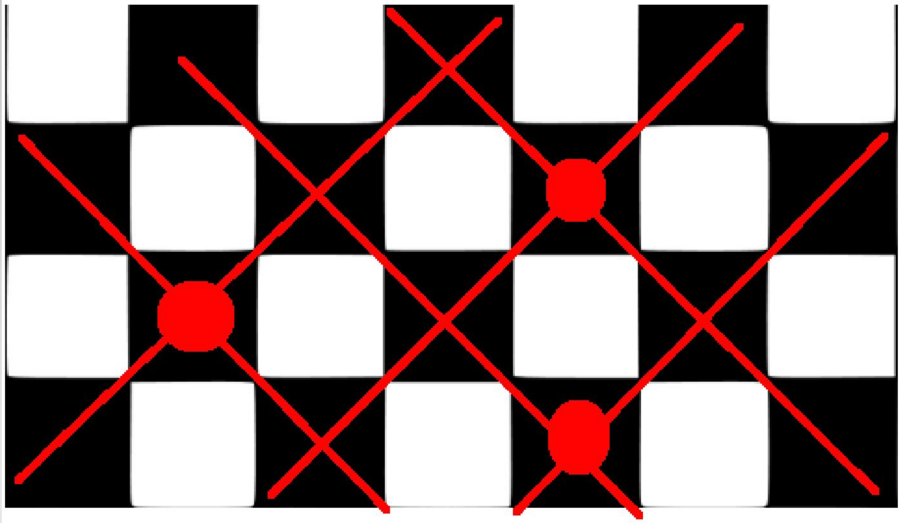
For a graph G, a *dominating set*, S, is defined to be a set $S \subseteq V$, where all vertices $v \in V - S$ are adjacent to at least one member of S. The vertices of a dominating set represents the squares where one would place rooks or bishops in order to attack every square on the chessboard. The number, $\gamma(G)$

maximum independence number, $B_0(G)$, is the maximum cardinality of an independently dominating set for a graph G. Independent sets are also known

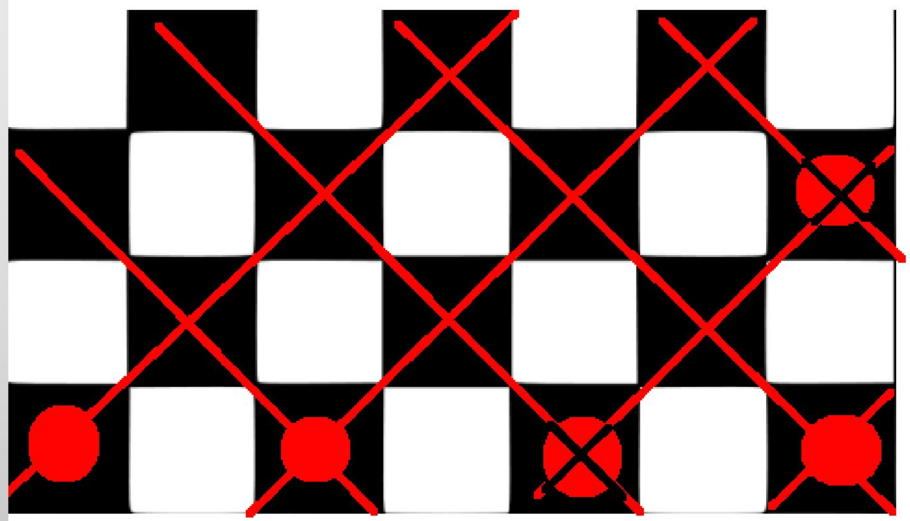
An *independently dominating set* is a subset $S \subseteq V$, where for all vertices $v \in S$, v is not adjacent to any other vertex in S and S dominates every square on the chessboard. The *minimum independence number*, $i(G)$, is the minimum cardinality of an independently dominating set for a graph G. The *maximum independence number*, $B_0(G)$, is the maximum cardinality of an

Don't be discouraged if you don't know how to play chess.
Domination has little to do with the actual game of chess

Not independent!!



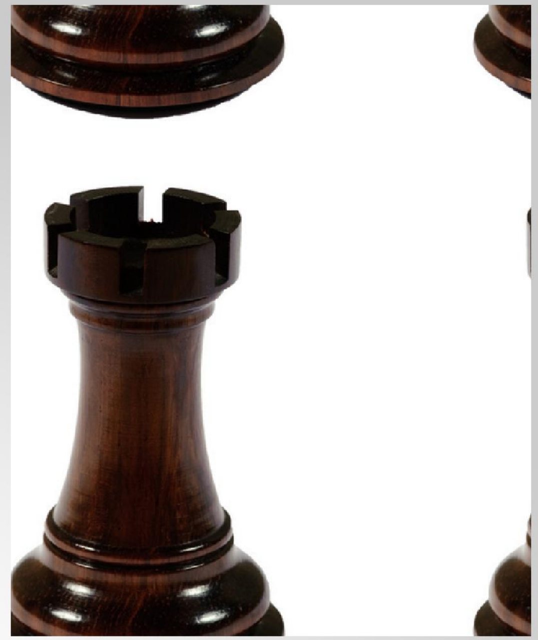
$\gamma(B_{4,7}) = 6 = i(B_{4,7})$



Not independent!!

Rook Domination

- Relatively simple compared to chessboard domination.
- In two dimensions, the domination value is the minimum dimension of the board.

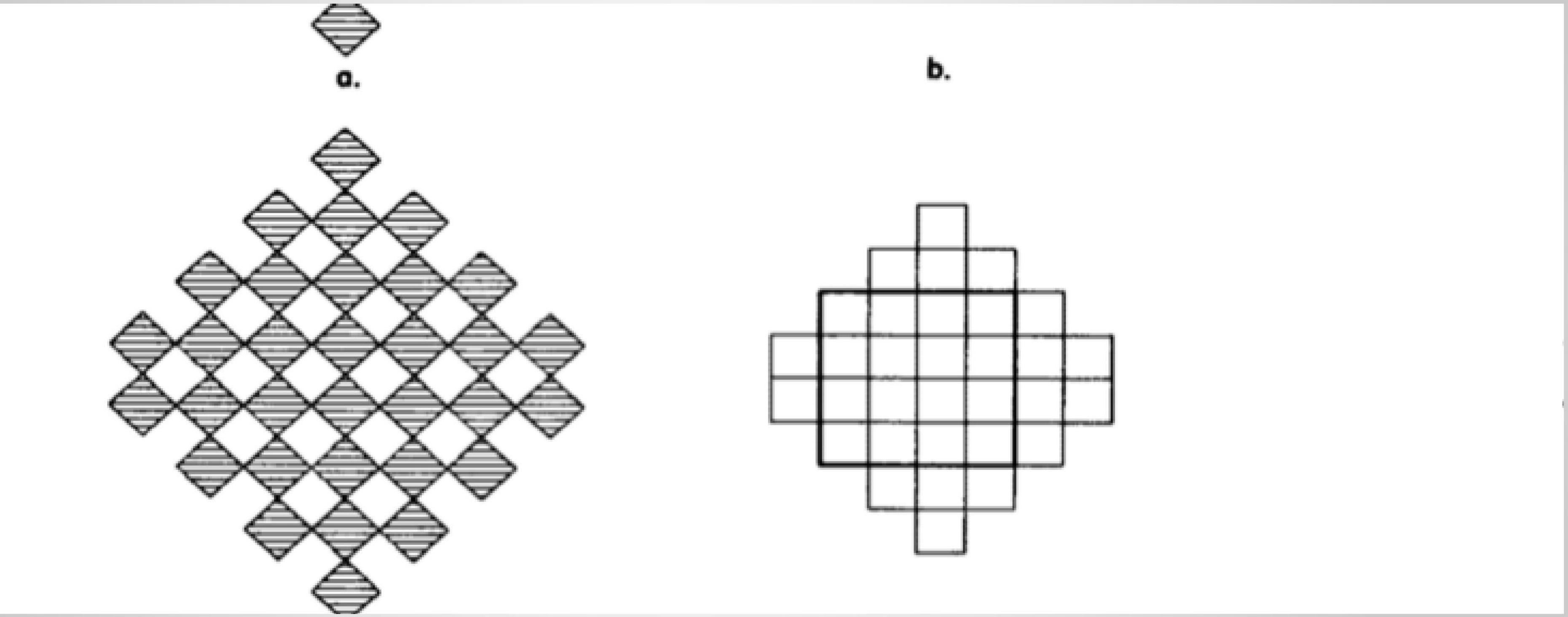


Theorem 2.1 $\gamma(R_{m,n}) = \gamma^t(R_{m,n}) = i(R_{m,n}) = B_0(R_{m,n}) = min\{m, n\}$ $\forall m, n \in \mathbb{N}$.

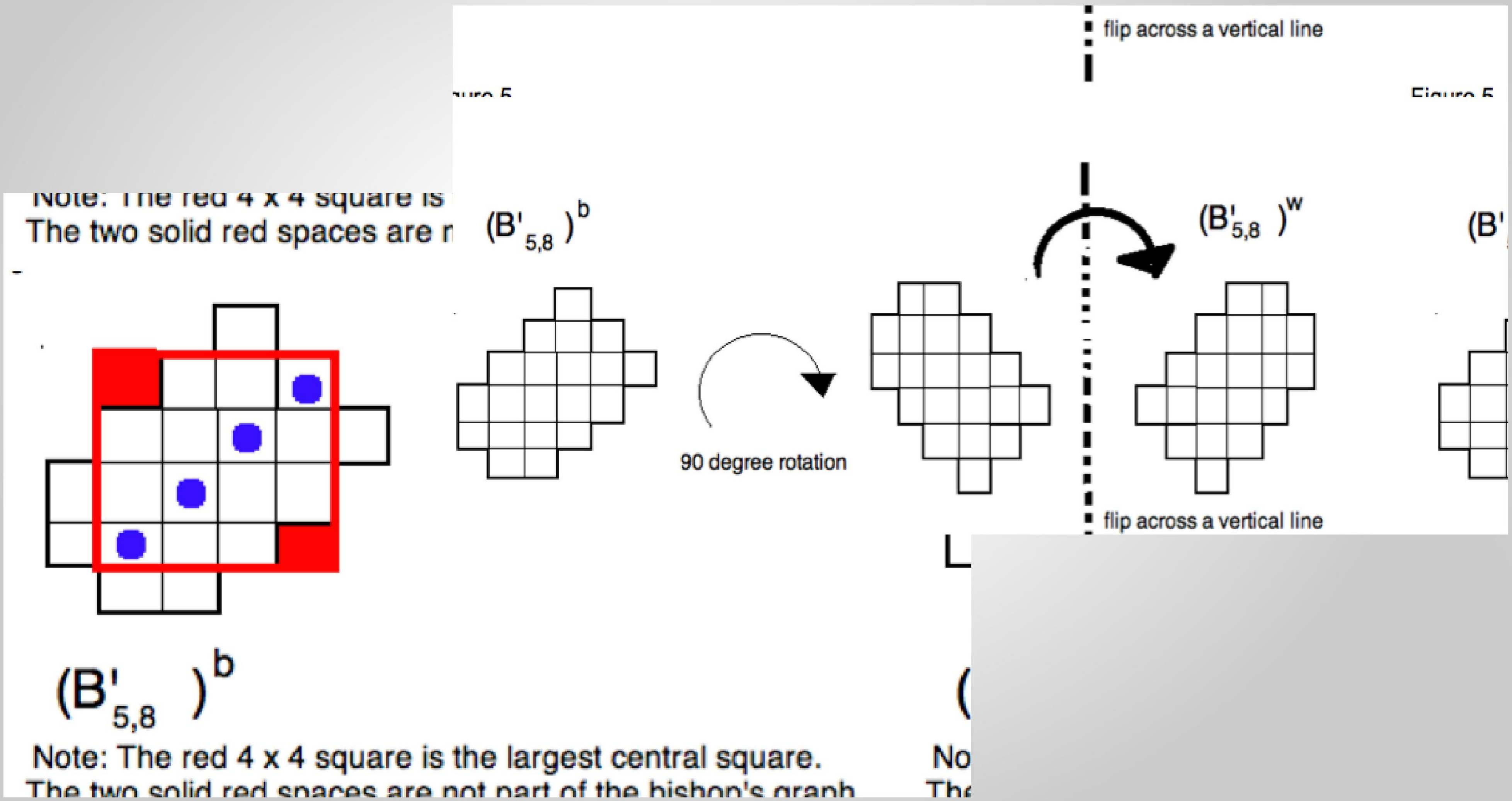
Bishop Domination

- Due to the diagonal motion of bishops, their domination is more exciting.
- Since these pieces move diagonally they can only move to squares of the same color.
- This means that we can think of the domination of the white squares and black squares separately.

Transformations of Bishops Graphs

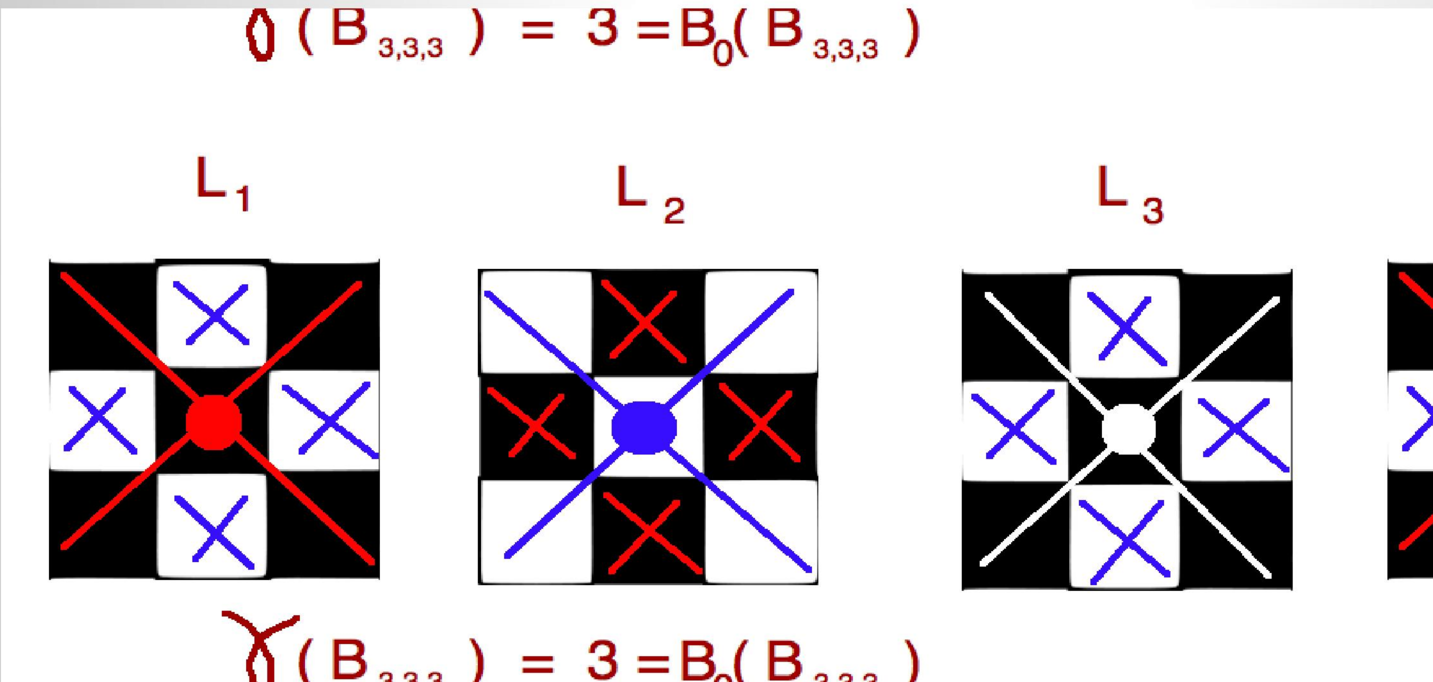


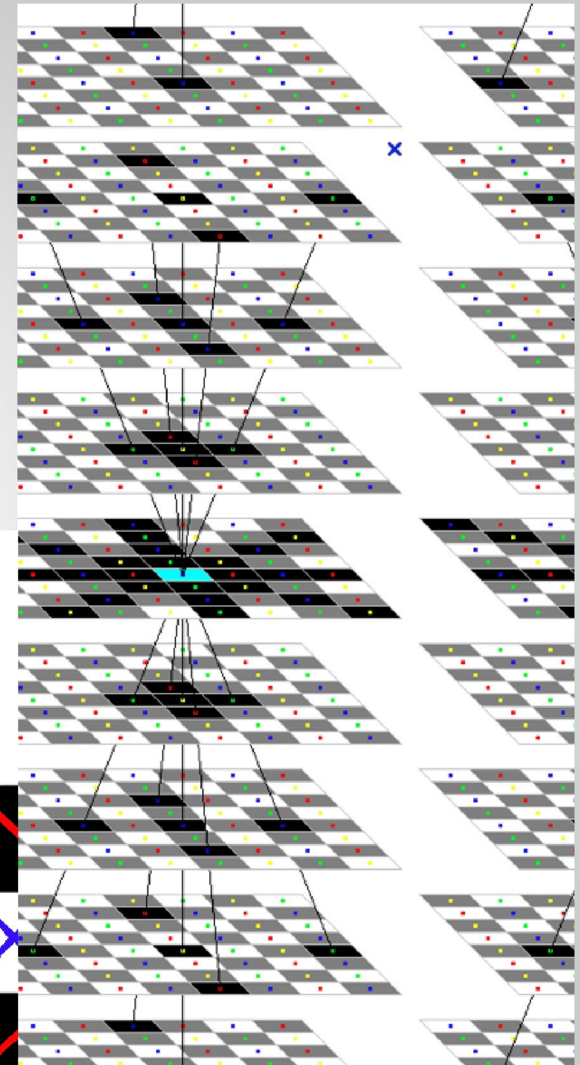
Black and White Subgraphs



Domination in three dimensions

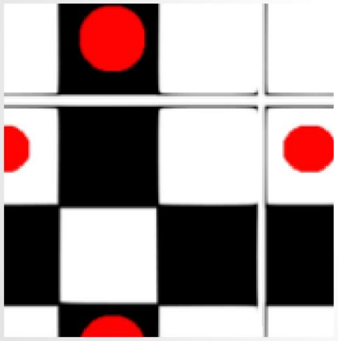
- Now we have arrays, (L1, L2 ... LN), of matrices to represent the spatial layers of the chess cubes.

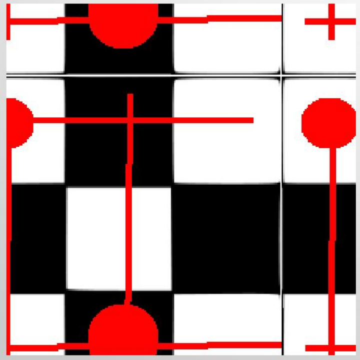




Domination tools

- I created a set of domination tools to visualize threatened squares
- The position matrix have non-zero values for spaces occupied by chesspieces
- Attack matrices have non-zero values in squares that are being threatened or attacked
- My domination tools are algorithms that take attack and position matrices as inputs and return whether or the arrangement of pieces is dominating or independently dominating.





```
     0        1        0

:itionMatrix =         Posi

     1        0        0
     0        0        0
     1        1        1

ackMatrix =           Attac

     1        1        1
     1        1        0
     1        1        1
```

How Matlab is furthering my research....

- I am automating the creation of attack matrices so I can find every arrangement of pieces. My data set will eventually include all three dimensional chess boards such that no side is more than 20 squares long.
- Once this data set is complete I can find the trends and prove them for all combinations of natural numbers.
- Matlab is also able to help me find the number of unique arrangements of pieces there are. This is an interesting piece of information, mathematically speaking...

Results and Next Steps

From my current data, it can be said that:

$$k(\gamma(B_{i,j})) \geq \gamma(B_{i,j,k}) \quad k($$

$$k(i(B_{i,j})) \geq i(B_{i,j,k}) \quad k($$

$$k(B_0(B_{i,j})) \geq B_0(B_{i,j,k}) \quad k($$

$$k(B_0(B_{i,j})) \geq B_0(B_{i,j,k}) \quad k($$

- **Next I need to prove this inductively, since my data set will only include boards with a maximum size length of 20.**
- **I will also prove theorems, so that for any i, j, k one could accurately know how many pieces make up the different domination constants**

Bibliography and Thanks for Listening

[3] A. M. Yaglom and I. M. Yaglom, Challenging Mathematical Problems with Elementary Solutions, Holden-Day, Inc., San Francisco, 1964.

References

[1] J. Eisenberg and A. Hallet, Chessboard Domination, 2011.

[2] DeMaio, J. and Faust W. (2009) http://science.kennesaw.edu/ jdemaio/ bishops%20domination%20torus%202%20col.pdf. Domination and Independence on the Rectangular Torus by Rooks and Bishops. Paper presented at the Proceedings of The 2009 International Conference on Foundations of Computer Science, Las Vegas, NV. Retrieved December 11, 2011.

[3] A. M. Yaglom and I. M. Yaglom, Challenging Mathematical Problems